Neuroeconomics
Macpen Evolution
A tale of Macpan life, conflict, sex, and death

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Abstract

Neuroeconomics aims to explain human decision-making and the capacity to consider several options and follow a course of action. In this problem statement, we have analyzed the effects of small individual decisions on something as massive as the population.

Figure 1: Artistic depiction of macpen by macpanthropologists

1 ENVIRONMENT DESIGN

1.1 Grid

Our simulation is run on a grid of dimensions $m \times n$,

$$G = \{1, 2, \ldots m\} \times \{1, 2, \ldots n\} = \{(x, y) : x \in [m], y \in [m]\}$$

where we denote by $[N] = \{1, 2, \ldots N\}$. The evolution of the Macpen is played out over a sequence of days indexed by $t = 0, 1, 2, \ldots$

1.2 Canteens

We chose a parameter, the number of canteens in a board, say c. We can interpret the number of canteens as the *abundance of resources* available to the Macpen.

1.2.1 Canteen Respawning mechanism

After every period of a fixed number of days, the canteen lifetime t_C, i.e. at $t = 0, t_C, 2t_C \ldots$, c canteens, $C_1, C_2 \ldots C_c$ are spawned on the grid. The locations of the canteens are chosen randomly uniformly i.e. each canteen chooses its coordinates using a discrete uniform distribution

$$x_i \sim \mathcal{U}\{1, m\}, y_i \sim \mathcal{U}\{1, n\}, \forall i \in [c]$$

Whenever a Macpan reaches a cell that contains a canteen, we assume that it is dispensed a food packet with ϕ food.

2 Macpen

We denote the population of macpen at any time t by $P(t)$. We start with a seed population of N macpen. The macpen are initially placed in the grid randomly, uniformly similar to the canteens.

2.1 Distribution

As in the problem statement the macpen have three distinct possible traits, which we interpret as follows:

1. *GRATEFUL*: We assume that such a macpan always shares any excess food that it had with a macpan that is dying due to insufficient food once the ghost gang has incurred its tax.
2. *UNGRATEFUL*: Such macpen are indifferent to the state of other macpen in the cell that it is currently in.
3. *TIT-FOR-TAT*: Such macpen make sharing decisions based on their history with the population. Each tit-for-tat macpan starts off with a probability $p = 1$ that it will share with a dying macpan. Whenever it experiences a situation where it might die, if it gets saved it updates the probability that it will help a dying macpan the next time it encounters one is updated as:

$$p \leftarrow \frac{\text{Number of macpen who share with it}}{\text{Total number of macpen in the cell}}$$

   This parameter represents the general tendency of macpen to share and subsumes within it multiple factors including but not limited to the proportion of grateful macpen, the relative

3 STRATEGY

3.1 Macpen Movement Strategies

1. *GREEDY STRATEGY*: The canteens are randomly spawned. The macpen move towards the nearest canteen.
2. *COPYCAT STRATEGY*: The canteens are randomly spawned and macpen copy (repeat the movements) of the most food-rich macpen.
3. *FOLLOW STRATEGY*: The canteens are fixed and the macpen attempts to reach the cell of the most food-rich macpen.
4. *SOURCE STRATEGY*: The canteens are fixed at a particular position, and macpen randomly move towards every direction.

3.2 Resource Allocation Strategies

1. *GOD*: Canteen possess unlimited amount of food.
2. *LIMITED FOOD*: Canteen possesses limited food in each of its spawn, and distributes less and less food as time passes.
3. *CAPITALISM*:Canteen doubles your food.
4. *COMMUNISM*: Each macpen leaves canteen with same amount of food.

3.3 Reproduction strategy

1. *BASE FOOD STRATEGY*:Macpen will reproduce once it has reached a baseline amount of food.
2. *PROBABILISTIC REPRODUCTION*: Macpen will reproduce with a certain probability which will be a increasing function wrt food.

3.4 Sharing strategy

1. *EQUALITY*: Excess food will be equally distributed among the dying members
2. *LUCK*: A dying member would get food until it reaches charity threshold. Remaining members would be left to die.

4 SIMULATION RESULTS REASONING

4.1 MODELLING CIVILIZATION

Under this section, a simulation of macpen was created. There was a single source of canteen fixed in location. Macpen moved randomly wit equal probability of going in each direction.
Under this simulation, 1000 macpen are placed in a grid of 10X10, and simulated for 20 time steps.

1. *ALL MACPEN ARE UNGRATEFUL*: In this case, we obtain the following graph:

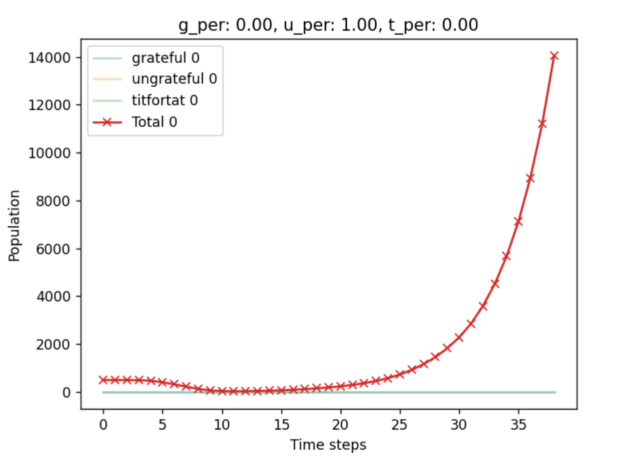

Figure 2: ALL MACPEN ARE UNGRATEFUL

On iterating this 100 times, we obtain the following statistics.
MEAN = 4983
STANDARD DEVIATION = 1447
MAXIMUM END RESULT = 10300
MINIMUM END RESULT = 2877

2. ALL MACPEN ARE GRATEFUL: In this case the following graph is obtained:

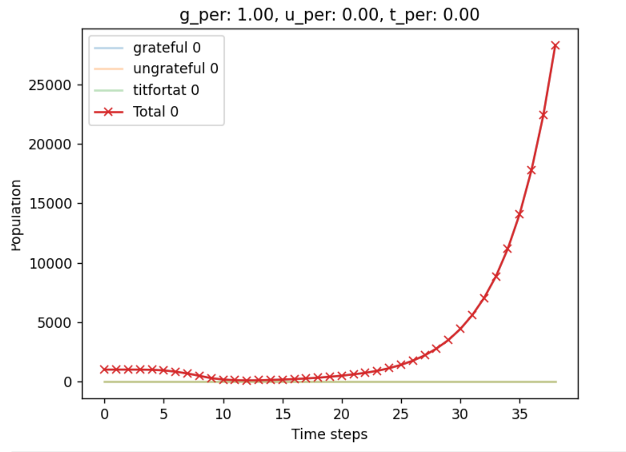

Figure 3: ALL MACPEN ARE GRATEFUL

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On iterating this 100 timelines, we obtain the following statistics, at the end of 20 turns.
MEAN = 5920
STANDARD DEVIATION = 2059
MAXIMUM END RESULT = 150720
MINIMUM END RESULT = 3092

3. *ALL MACPEN ARE TIT FOR TAT*: In this case, we obtain the following graph:

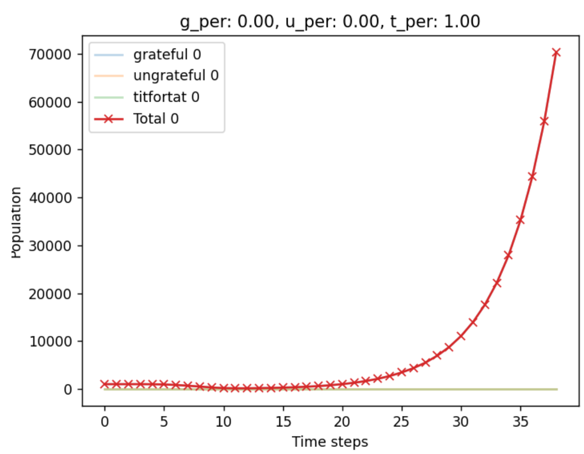

Figure 4: ALL MACPEN ARE TIT-FOR-TAT

On iterating this 100 times, we obtain the following statistics.
MEAN = 5533
STANDARD DEVIATION = 2508
MAXIMUM END RESULT = 19554
MINIMUM END RESULT = 2830

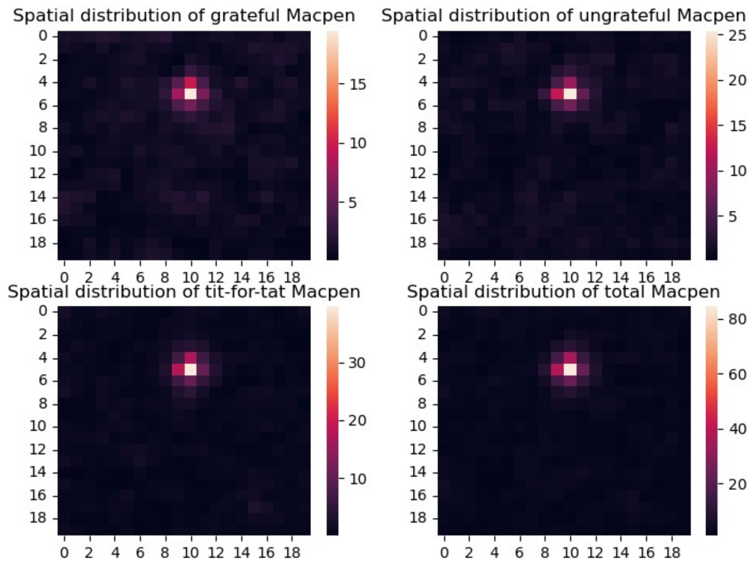

Figure 5: SPATIAL DISTRIBUTION OF MACPEN

RESULT ANALYSIS: The following approach tells that ultimately in this approach, there is a short term population decline, followed by an exponential growth. Ultimately the trait that does not matter in qualitative sense, in the post stability period, every trait grows in number. However trait does matter in quantitative sense. Grateful Macpen have the highest average while the ungrateful have the least. This can be explained as Macpen are allowed to go food-less in some sense. They are allowed to go in a *debt*, of sorts, giving them most flexibility. Interestingly the mean of tit for tat is approximately the mean of Grateful and ungrateful ones. Tit for Tat is very spread out distribution having the largest standard deviation, and highest peaks and lows.
The heat map of a typical Macpen spatial distribution is given aove.
The macpen are closely concentrated near the source, something which intution suggests. A reasoning for this can be stated like this - *Suppose a macpen which food f is at a distance greater than f from the canteen, then in no condition can it survive.*
Ultimately there is a radius for each macpen, which it can not escape.

God does favour the grateful

4.2 MODELLING EXTINCTION EVENTS

Under this simulation. There is again a single source of canteen. However after every *t* days that canteen relocates to a separate location.
Under this approach, the following graphs is obtained:

1. *ALL ARE GRATEFUL*:

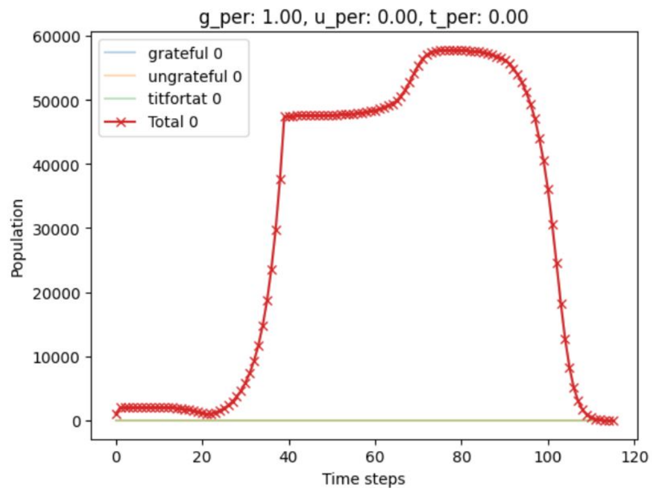

Figure 6: ALL MACPEN ARE GRATEFUL

2. *ALL ARE UNGRATEFUL*:

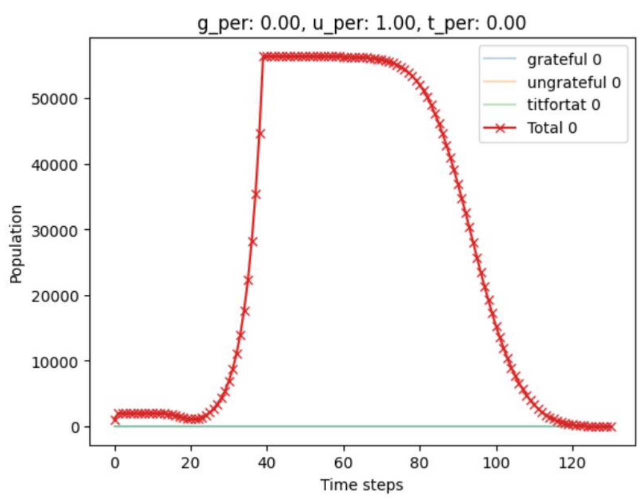

Figure 7: ALL MACPEN ARE UNGRATEFUL

3. *ALL FOLLOW TIT FOR TAT*:

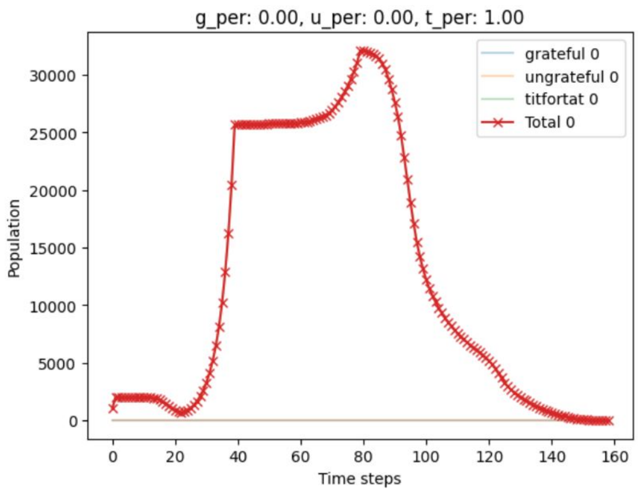

Figure 8: ALL MACPEN ARE TIT-FOR-TAT

RESULT ANALYSIS: From the graph, it is observed that

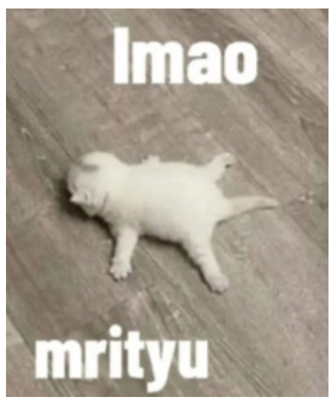

Figure 9: Mrityu happens

Ungrateful populations are more susceptible to extinction. Ungrateful populations have a feature, that it grows faster but also collapses faster than the others.
Grateful population do not really grow or collapse very fast. There curves are much smoother than the rest. Tit for Tat is a compromise between the two.
Another property which can be learned by this is that population is at its most vulnerable when it is in the reproduction phase. A population can still sustains its number during an *extinction event*, but numbers rapidly fall if it occurs during the *exponential growth phase*.

4.3 MODELLING CAPITALISM

Under this section of macpen simulation, there was a single source of canteen fixed in location. Capitalism approach will double the food of each macpen going in. Macpen moved randomly with equal probability of going in each direction.
Under this simulation, 1000 macpen are placed in a grid of 10×10, and simulated for 20 timesteps.

1. *ALL MACPEN ARE UNGRATEFUL*: In this case, we obtain the following graph:

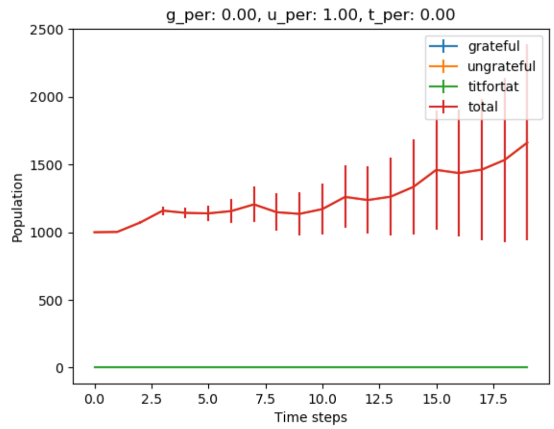

Figure 10: ALL MACPEN ARE UNGRATEFUL

On iterating this 100 times, we obtain the following statistics.
MEAN = 1662
STANDARD DEVIATION = 728
MAXIMUM END RESULT = 5571
MINIMUM END RESULT = 776

2. ALL MACPEN ARE GRATEFUL: In this case the following graph is obtained:

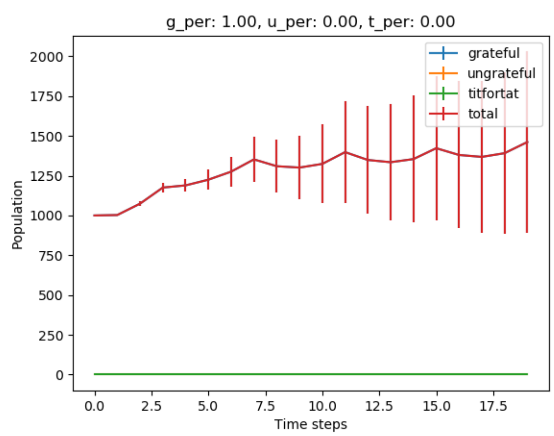

Figure 11: ALL MACPEN ARE GRATEFUL

On iterating this 100 timelines, we obtain the following statistics, at the end of 20 turns.
MEAN = 1461
STANDARD DEVIATION = 571
MAXIMUM END RESULT = 3566
MINIMUM END RESULT = 664

3. *ALL MACPEN ARE TIT FOR TAT*: In this case, we obtain the following graph:

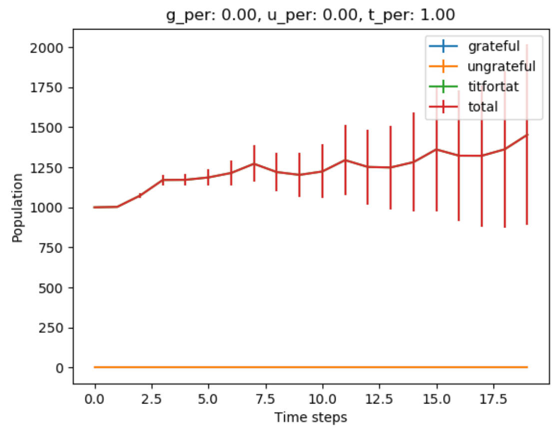

Figure 12: ALL MACPEN ARE TIT-FOR-TAT

On iterating this 100 times, we obtain the following statistics.
MEAN = 1452
STANDARD DEVIATION = 565
MAXIMUM END RESULT = 4162
MINIMUM END RESULT = 726

4. *EQUAL DISTRIBUTION*: In this case, equal number of macpens for each trait are initially present, we obtain the following graph:

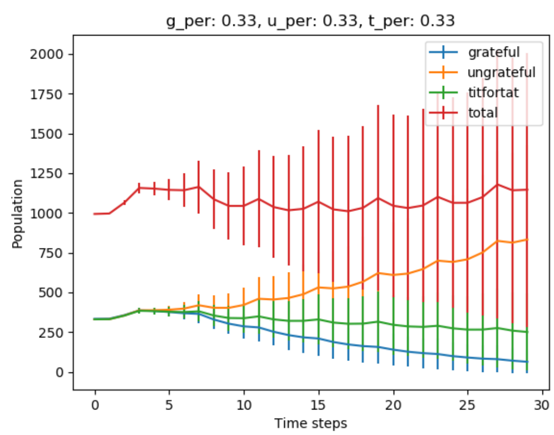

Figure 13: EQUAL APPROACH

On iterating this 100 times, we obtain the following statistics.
MEAN = 1145
STANDARD DEVIATION = 866
MAXIMUM END RESULT = 4096
MINIMUM END RESULT = 157

Food distribution:

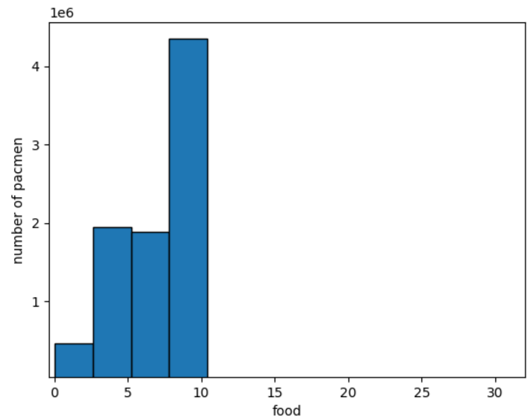

Figure 14: FOOD vs PACMAN DISTRIBUTION

RESULT ANALYSIS: Unlike the god approach, Ungrateful is by far the best performing trait. This can be easily understood by the reasoning of returns. The more food a macpen accumulates, the more food he receives from canteen. Interestingly the trend for tit for tat reverses in this approach. They have the much more compact set of results, resembling the mean between ungrateful and grateful, while having the least standard deviation.
When they had equal numbers, it was observed that the grateful had the most decline in the population while Ungrateful had an ascent. It also resembles a steady state system, with the average population just increasing by 10 percent over 20 time steps. The model of capitalism underperforms with respect to the model of 'GOD'.

4.4 MODELLING COMMUNISM

Under this section of macpen simulation, there was a single source of canteen fixed in location. Communist approach will be that a macpan entering a cell will leave with the same amount of food. Macpen moved randomly with equal probability of going in each direction.
Under this simulation, 1000 macpen are placed in a grid of 10X10, and simulated for 20 time steps.

1. *BARE MINIMUM*: In this case, equal number of each trait macpen were created. They will leave canteen with bare minimum possible food.

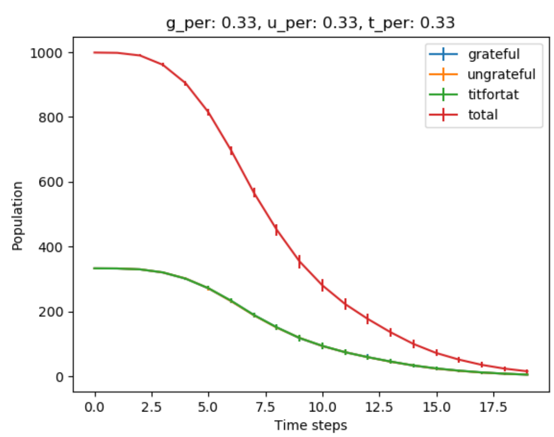

Figure 15: BARE MINIMUM

On iterating this 100 times, we obtain the following statistics.
MEAN = 15.4
STANDARD DEVIATION = 5.9
MAXIMUM END RESULT = 5
MINIMUM END RESULT = 33

2. *EXCESSIVE COMMUNISM*: In this case, equal number of each trait macpen were created. They will leave canteen with enough food to reproduce.

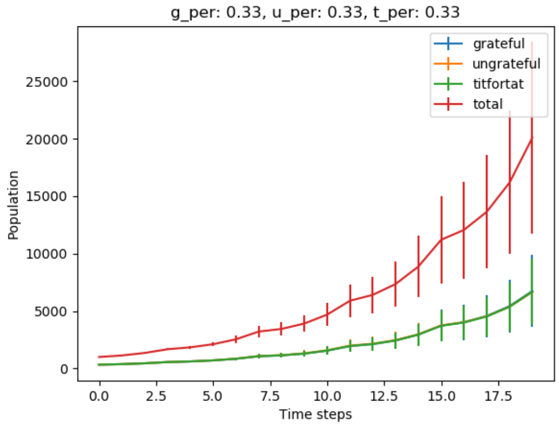

Figure 16: BARE MINIMUM

On iterating this 100 times, we obtain the following statistics.
MEAN = 20079
STANDARD DEVIATION = 8377
MAXIMUM END RESULT = 60929
MINIMUM END RESULT = 7751

Food distribution:

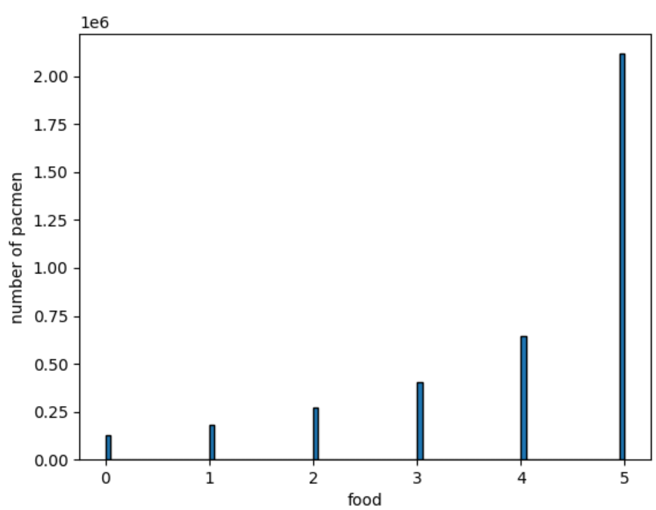

Figure 17: FOOD vs PACMAN DISTRIBUTION

RESULT ANALYSIS: These were very difficult to balance approaches. It is essentially impossible to have reproduction in bare minimum approach, and very difficult to control it in excessive communism approach. Not much can be made of the population statistic, but an interesting property emerged in the trait aspect. There was no visible trait difference in population.
Essentially all traits became equal.

5 MISCELLANEOUS

5.1 Movement: GREEDY APPROACH

Claim 1. *Macpen in the same cell at time $t = t_0$, will remain in the same cell for all $t > t_0$.*

Proof. Let $M_1, M_2 \dots M_n$ denote the Macpen in some cell. Clearly, the nearest canteen and hence, the direction of motion in the next step is the same for each M_i, $i \in [n]$. Therefore by mathematical induction, the preceding claim holds. ☐

Claim 2. *Macpen will essentially be huddled into a single cell.*

Proof. Macpen can never 'disperse' from a single cell. If canteen appears at cell $a_{t,t}$, then Macpen at $a_{t,(t+1)}, a_{(t+1),t}, a_{(t-1),t}, a_{t,(t-1)}$, will assemble at cell a_{tt}.
Hence unification is possible, dispersion is not. For a random canteen spawn algorithm, Macpen will eventually unify in a single cell. ☐

Claim 3. *Macpen can never have food greater than the reproduction threshhold.*

Proof. If a Macpan has food greater than reproduction threshhold, then it must divide in accordance to our environment rules. ☐

Claim 4. *For the goal of maximizing population, even the grateful macpen should not share.*

Proof. Assume a simple Mathematical model, with two Macpen in a single cell. Let us assume them to be M_1, M_2. Suppose M_1 has f food, and M_2 has 0 (nearing death). Since *share* is enabled while they are in the same cell, they will remain in the same cell forever.
There are two possible cases, M_1 shares t *units* of food, and M_1 lets M_2 die.
Assumptions of our model:
a) The appearance of canteen in enough for Macpen to divide.
b) Canteen changes location at every step.
c) M_1 will never share more than half his food

Case 1: M_1 shares t *units* of food

$$\mathbb{P}(\text{ atleast one survives}) = 1 - \mathbb{P}(\text{ Both die})$$
$$\mathbb{P}(\text{ both die}) = \mathbb{P}(\text{ canteen does not appear for f-t steps}) = (1-p)^{f-t}$$
$$\mathbb{P}(\text{ atleast one survives}) = 1 - (1-p)^{f-t}$$

Case 2: Let us consider M_1 lets M_2 die.

$$\mathbb{P}(M_1 \text{ survives}) = 1 - \mathbb{P}(M_1 \text{dies})$$
$$\mathbb{P}(M_1 \text{ dies}) = \mathbb{P}(\text{ canteen does not appear for f steps}) = (1-p)^f$$
$$\mathbb{P}(M_1 \text{ survives}) = 1 - (1-p)^f$$
$$1 - (1-p)^{f-t} < 1 - (1-p)^f$$

Therefore the optimal strategy to maximize population is to not share food. ☐

For the greedy approach we obtain the following spatial distribution.

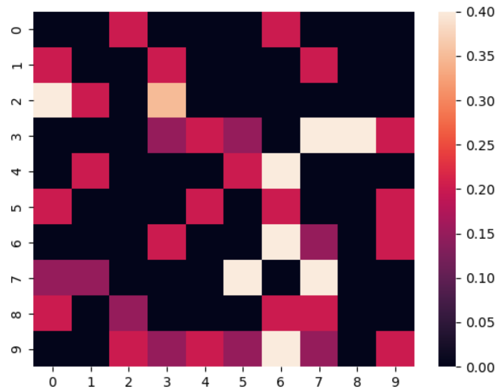

Figure 18: CANTEEN DISTRIBUTION

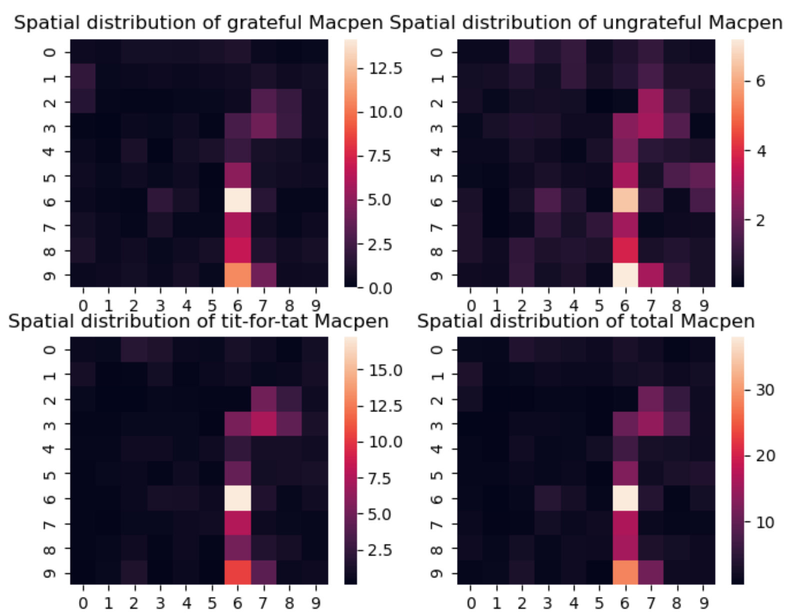

Figure 19: SPATIAL DISTRIBUTION OF MACPEN

5.2 Movement: CONWAY'S GAME OF LIFE APPROACH

This movement policy is inspired by the game of life proposed by John Conway. In this movement policy, each agent is assigned a direction to move randomly, uniformly with probability $\frac{1}{4}$. Further, when an agent reproduces, as in life, its child moves away from the parent. That is, if an agent is moving in the direction $v \in \mathbb{R}^2$, then its child will start moving in the direction given by $-v$.
For this we obtain the following spatial distribution.

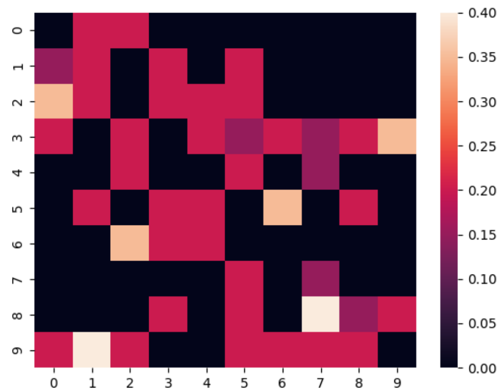

Figure 20: CANTEEN DISTRIBUTION

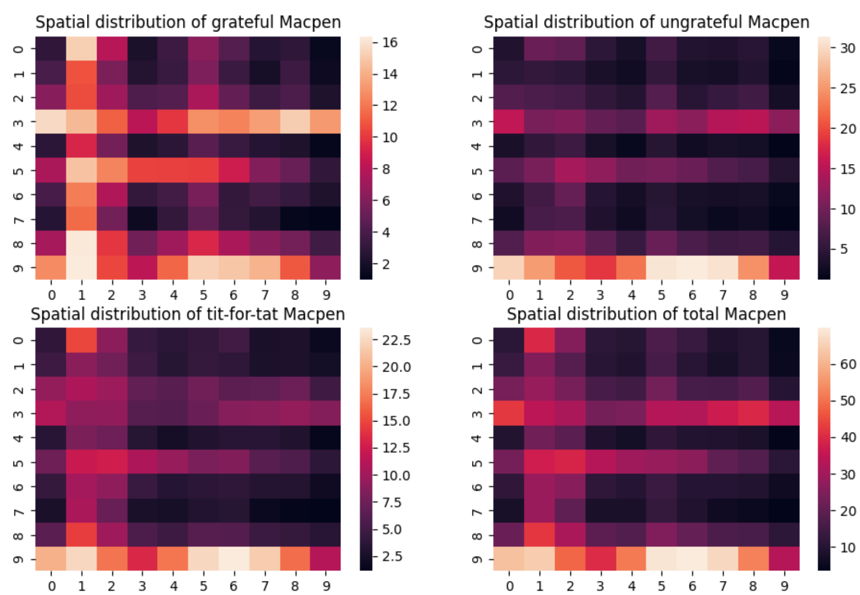

Figure 21: SPATIAL DISTRIBUTION OF MACPAN

5.3 Tribal Warfare

We simulate the environment in a way that:

- Ungrateful macpen kill all the grateful macpen present in the cell if they have the majority in it.
- Grateful macpen kill all the ungrateful macpen present in the cell if they have the majority in it.

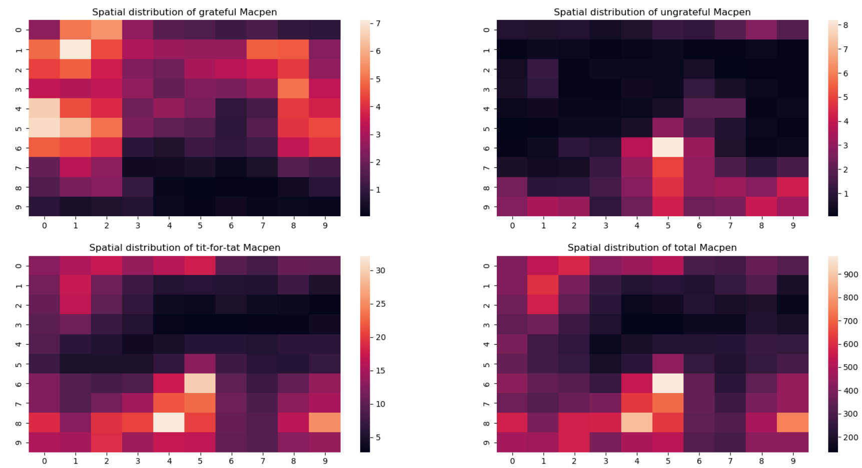

Figure 22: SPATIAL DISTRIBUTION OF MACPEN

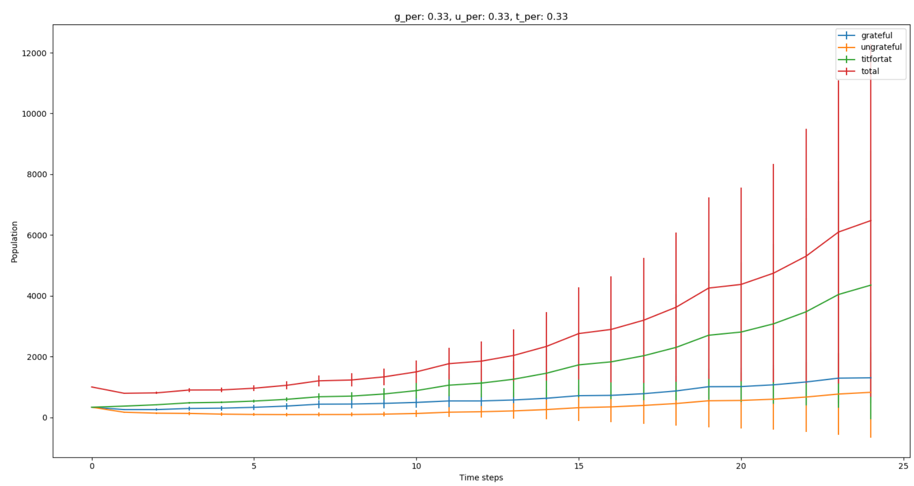

Figure 23: POPULATION GROWTH OF MACPEN

RESULT:

- Ungrateful macpen always die in the end. Since they are killed whenever in minority by the grateful ones, and they do not share food among themselves, they eventually get extinct.
- Tit-for-tat macpen always dominates in the end. Since they are not at war with any community, their evolution is natural and sustained eventually in the population.

5.4 Left-Right

We split the grid into two halves along the vertical line of symmetry. The *left* half, henceforth called the communist empire, follows communist food distribution and the *right* half, henceforth referred to as the capitalist state follows capitalist distribution. The following were observed:

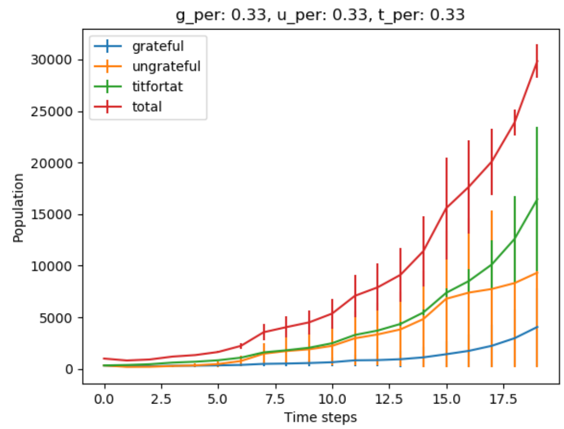

Figure 24: Left Right POPULATION DISTRIBUTION

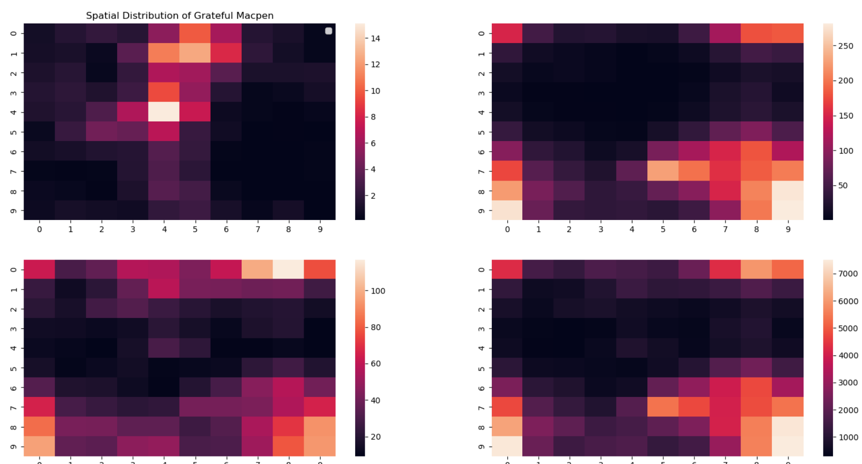

Figure 25: Left Right Spatial Distribution averaged over multiple timelines

RESULT:

- It's clear from the figure that capitalism is supported by Ungrateful agents, while communism + capitalism is supported Grateful ones, with Tit-for-Tat tending towards capitalism.
- The population growth displays a rise in capitalist based community, that includes primarily Ungrateful and Tit-for-Tat.